Question 1

$$\frac{b}{a-p} = -m \Rightarrow p = a + \frac{b}{m}$$ **M1 A1**

$$\frac{b-q}{a} = -m \Rightarrow q = b + am$$ **M1 A1**

Sum of distances of *P* and *Q* from the origin is

$$p + q = a + b + \left(am + \frac{b}{m}\right)$$ **B1**

Differentiate to find value of *m* for which least value occurs:

$$\frac{d}{dm}(p+q) = a - \frac{b}{m^2}$$ **M1 A1**

$$a - \frac{b}{m^2} = 0 \Rightarrow m = \sqrt{\frac{b}{a}}$$ **M1 A1**

Minimum value is

$$a + b + 2\sqrt{ab} = \left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^2$$ **M1 A1**

The minimum distance will be obtained when

$$p^2 + q^2 = \left(a + \frac{b}{m}\right)^2 + (b + am)^2$$ **B1**

is a minimum.

Differentiate to find value of *m* for which least value occurs:

$$\frac{d}{dm}(p^2 + q^2) = -\frac{2b}{m^2}\left(a + \frac{b}{m}\right) + 2a(b + am)$$ **M1 A1 A1**

$$-\frac{2b}{m^2}\left(a + \frac{b}{m}\right) + 2a(b + am) = 0 \Rightarrow m = \sqrt[3]{\frac{b}{a}}$$ **M1 A1**

Minimum value of $p^2 + q^2$ is

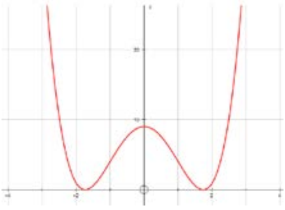
$$\left(a + a^{\frac{1}{3}}b^{\frac{2}{3}}\right)^2 + \left(b + a^{\frac{2}{3}}b^{\frac{1}{3}}\right)^2 = \left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^3$$ **M1 A1**

Minimum distance is

$$\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$$ **A1**

Question 2

(i)	$y = (x^2 - 3)^2$	**G1 (shape)**
	Minimum points are at $(\pm\sqrt{3}, 0)$	**G1 (minima)**
	Maximum point is at $(0, 9)$	**G1 (maximum)**



(a)	$n = 0$ for $b > 9$	**B1**
(b)	$n = 1$ is never true	**B1**
(c)	$n = 2$ for $b = 9$ and $b < 0$	**B1**
(d)	$n = 3$ for $b = 0$	**B1**
(e)	$n = 4$ for $0 < b < 9$	**B1**

(ii)

$$\frac{dy}{dx} = 4x^3 - 12x + a$$

$$\frac{d^2y}{dx^2} = 12x^2 - 12$$ **B1**

They can only both be equal to 0 if $x = \pm 1$ **A1**

$$a = \pm 8$$ **A1**

Find stationary points of $y = x^4 - 6x^2 \pm 8x$

$$\frac{dy}{dx} = 4x^3 - 12x \pm 8$$

$$4x^3 - 12x \pm 8 = 0 \Rightarrow (x \mp 1)^2(x \pm 2) = 0$$ **M1 A1**

Stationary points are

$$(\mp 2, 24) \text{ (minimum)}$$ **A1 A1**

$$(\pm 1, 3) \text{ (minimum)}$$

0 roots if $b > 24$ **B1**

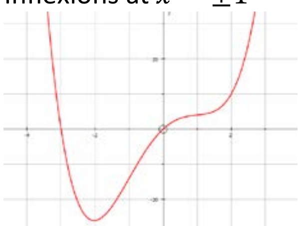
1 root if $b = 24$ **B1**

2 roots if $b < 24$ **B1**

if correctly solved for just one case, award all but one mark.

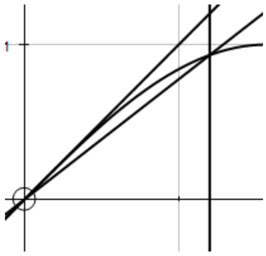
(iii) Minimum point for some negative value of x **G1**

Inflexions at $x = \pm 1$ **G1**



Question 3

(i)



B1

Adding the line $x = b$ shows that the area under the curve $y = \sin x$ is between the areas of two triangles.

Area of large triangle

$$\frac{1}{2}b^2$$

A1

Area of small triangle

$$\frac{1}{2}b\sin b$$

A1

Area under curve

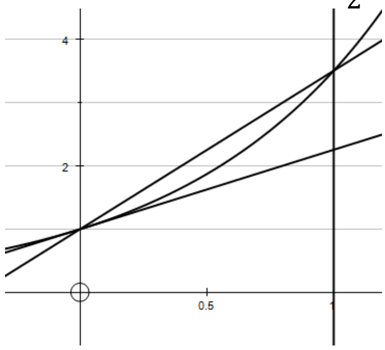
$$\int_0^b \sin x\,dx = 1 - \cos b$$

M1 A1

$$1 - \frac{1}{2}b^2 < \cos b < 1 - \frac{1}{2}b\sin b$$

M1 A1

(ii)



The area under the curve is between the areas of the two trapezia

Area of larger trapezium

$$\frac{1}{2}(1)(a+1)$$

A1

Area of smaller trapezium:

The gradient of the tangent needs to be found:

M1 A1

$$\frac{d}{dx}(a^x) = a^x \ln a$$

The gradient of the tangent at the origin is $\ln a$.

B1

The tangent passes through the point $(1, 1 + \ln a)$.

Area of trapezium

$$\frac{1}{2}(1)(1 + 1 + \ln a)$$

A1

Area under curve

$$\int_0^1 a^x\,dx = \left[\frac{1}{\ln a}a^x\right]_0^1$$

M1 A1

$$= \frac{1}{\ln a}(a-1)$$

$$\frac{1}{2}(2 + \ln a) < \frac{1}{\ln a}(a-1) < \frac{1}{2}(a+1)$$

B1

$$\frac{1}{\ln a}(a-1) < \frac{1}{2}(a+1) \Rightarrow \frac{2(a-1)}{a+1} < \ln a$$ **A1**

As $\ln a > 0$:

$$\frac{1}{2}(2+\ln a) < \frac{1}{\ln a}(a-1) \Rightarrow (\ln a)^2 + 2\ln a < 2a-2$$ **M1 A1**

$$(\ln a + 1)^2 < 2a - 1$$

$$\ln a + 1 < \sqrt{2a-1}$$ **A1**

Putting the two together:

$$\frac{2(a-1)}{a+1} < \ln a < -1 + \sqrt{2a-1}$$ **A1**

Question 4

$$\frac{dy}{dx} = -\frac{1}{2}x^{-2}$$ **A1**

Tangents at *P* and *Q* are

$$2p^2y + x = 2p$$ **M1 A1**

$$2q^2y + x = 2q$$

At *T*:

$$(p^2 - q^2)y = p - q \Rightarrow y = \frac{1}{p+q}$$ **M1 A1 A1**

$$(q^2 - p^2)x = 2pq(q - p) \Longrightarrow x = \frac{2pq}{p+q}$$

Coordinates of *T* are:

$$\left(\frac{2pq}{p+q}, \frac{1}{p+q}\right)$$ **A.G.**

Gradients of normal at *P* and *Q* are p^2 and q^2. **B1**

Normals at *P* and *Q* are

$$y - 2p^2x = \frac{1}{2p} - 2p^3$$ **M1 A1**

$$y - 2q^2x = \frac{1}{2q} - 2q^3$$

At *N*: **M1 A1**

$$2(q^2 - p^2)x = (q - p) + 2(q^3 - p^3)$$

Since $q^3 - p^3 \equiv (q - p)(q^2 + pq + p^2)$: **M1**

$$2(p + q)x = 1 + 2q^2 + 2pq + 2p^2$$

Since $1 = 2pq$:

$$(p + q)x = q^2 + 2pq + p^2$$ **M1 A1**

$$x = p + q$$

Substituting to get y:

$$y = 2p^2(p + q) + \frac{1}{2p} - 2p^3 = p + q$$ **M1 A1**

So *N* has coordinates $(p + q, p + q)$ **A1**

N clearly lies on the line $y = x$. **A1**

When $2pq = 1$, *T* has coordinates $\left(\frac{1}{p+q}, \frac{1}{p+q}\right)$, which is also on the line $y = x$.

The distances from the origin are $\sqrt{2}(p + q)$ and $\frac{\sqrt{2}}{p+q}$ **B1**

The product of the distances is always equal to 2. **B1**

Question 5

$$S = \int_0^{\frac{1}{4}\pi} \sin 2x \ln \cos x \, dx$$

Integrate by parts:

$$\frac{d}{dx}(\ln \cos x) = -\tan x \qquad \textbf{M1 A1}$$

$$\int \sin 2x \, dx = -\frac{1}{2}\cos 2x \qquad \textbf{A1}$$

Therefore

$$S = \left[-\frac{1}{2}\cos 2x \ln \cos x\right]_0^{\frac{1}{4}\pi} - \frac{1}{2}\int_0^{\frac{1}{4}\pi} \cos 2x \tan x \, dx \qquad \textbf{A1}$$

$$S = -\frac{1}{2}\int_0^{\frac{1}{4}\pi} \cos 2x \tan x \, dx$$

Using $\cos 2x \equiv 2\cos^2 x - 1$:

$$\int_0^{\frac{1}{4}\pi} \cos 2x \tan x \, dx = \int_0^{\frac{1}{4}\pi} \sin 2x - \tan x \, dx \qquad \textbf{M1 A1}$$

$$S = -\frac{1}{2}\left[-\frac{1}{2}\cos 2x + \ln \cos x\right]_0^{\frac{1}{4}\pi}$$

$$= \frac{1}{2}\left(\ln\sqrt{2} - \frac{1}{2}\right) \qquad \textbf{M1 A1 A.G.}$$

$$= \frac{1}{4}(\ln 2 - 1)$$

$$C = \int_0^{\frac{1}{4}\pi} \cos 2x \ln \cos x \, dx$$

Integrate by parts:

$$\frac{d}{dx}(\ln \cos x) = -\tan x$$

$$\int \cos 2x \, dx = \frac{1}{2}\sin 2x \qquad \textbf{A1}$$

Therefore

$$C = \left[\frac{1}{2}\sin 2x \ln \cos x\right]_0^{\frac{1}{4}\pi} + \frac{1}{2}\int_0^{\frac{1}{4}\pi} \sin 2x \tan x \, dx \qquad \textbf{A1}$$

$$C = -\frac{1}{2}\ln\sqrt{2} + \int_0^{\frac{1}{4}\pi} \sin^2 x \, dx \qquad \textbf{A1}$$

$\sin^2 x \equiv \frac{1}{2}(1-\cos 2x)$:

$$C = -\frac{1}{2}\ln\sqrt{2} + \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{1}{4}\pi} \qquad \textbf{M1 A1}$$

$$C = -\frac{1}{2}\ln\sqrt{2} + \frac{1}{8}\pi - \frac{1}{4} = \frac{1}{8}(\pi - \ln 4 - 2) \qquad \textbf{A1 A.G.}$$

$\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi}(\cos 2x + \sin 2x)\ln(\cos x + \sin x)dx$:

$$\cos x + \sin x \equiv \sqrt{2}\cos\left(x - \tfrac{1}{4}\pi\right)$$ **M1 A1**

Setting $u = x - \frac{1}{4}\pi$ the integral becomes:

$$\int_0^{\frac{1}{4}\pi} (-\sin 2u + \cos 2u)\ln\sqrt{2}\cos u\, du$$ **M1 A1**

Using $\ln\sqrt{2}\cos u = \frac{1}{2}\ln 2 + \ln\cos u$ gives:

$$\frac{1}{2}\ln 2\int_0^{\frac{1}{4}\pi} (\cos 2u - \sin 2u)\, du - S + C$$

$$= \tfrac{1}{2}\ln 2\left(\tfrac{1}{2} - \tfrac{1}{2}\right) - S + C = \tfrac{1}{8}\pi - \tfrac{1}{2}\ln 2$$ **M1 A1**

Alternative for part (i)

$$S = \int_0^{\frac{1}{4}\pi} \sin 2x \ln\cos x\, dx$$

Use $\sin 2x \equiv 2\sin x\cos x$:

$$S = \int_0^{\frac{1}{4}\pi} 2\sin x\cos x\ln\cos x\, dx$$ **B1**

Substitute $u = \cos x$:

$$\frac{du}{dx} = -\sin x$$

$$S = \int_1^{\frac{\sqrt{2}}{2}} -2u\ln \mathrm{u}\, du$$ **M1 A1**
A1 (limits)

Integrating by parts:

$$S = [-u^2\ln u]_1^{\frac{\sqrt{2}}{2}} + \int_1^{\frac{\sqrt{2}}{2}} u du$$ **M1 A1 (integrate by parts)**

$$S = -\frac{1}{2}\ln\frac{\sqrt{2}}{2} + \frac{1}{4} - \frac{1}{2}$$

$$S = \frac{1}{4}(\ln 2 - 1)$$ **M1 A1 (simplify)**

Question 6

The height of the pole is

$$CD\tan\varphi = AD\tan\alpha = BD\tan\beta$$ **B1**

Triangle ABC is right angled **B1**

Since triangles *ADC* and *CDB* are similar:

$$\frac{AD}{CD} = \frac{CD}{BD}\ (so\ CD^2 = AD \times BD)$$ **M1 A1**

Therefore

$$(CD\tan\varphi)^2 = (AD\tan\alpha) \times (BD\tan\beta)$$ **M1 A1 A.G.**

$$\tan^2\varphi = \tan\alpha\tan\beta\ \ (and\ so\ \cot^2\varphi = \cot\alpha\cot\beta)$$

$$LHS = \cos p\cos q\left(\tfrac{1}{2}(1-\cos(p+q))\right) - \sin p\sin q\left(\tfrac{1}{2}(1+cos(p+q))\right)$$ **M1 A1**

$$= \tfrac{1}{2}(\cos p\cos q - \sin p\sin q) - \tfrac{1}{2}cos(p+q)(\cos p\cos q + \sin p\sin q)$$ **M1**

$$= \tfrac{1}{2}\cos(p+q) - \tfrac{1}{2}\cos(p+q)\cos(p-q) = RHS$$ **M1 A1 A.G.**

If p and q are positive and $p + q \le \frac{1}{2}\pi$, then

$$\cos(p+q) \ge 0$$ **B1 B1**

$$1 - \cos(p-q) \ge 0$$

Therefore,

$$\cos p\cos q\sin^2\tfrac{1}{2}(p+q) - \sin p\sin q\cos^2\tfrac{1}{2}(p+q) \ge 0$$ **A1**

$$\cos p\cos q\sin^2\tfrac{1}{2}(p+q) \ge \sin p\sin q\cos^2\tfrac{1}{2}(p+q)$$

Since $\sin p$, $\sin q$ and $\sin\frac{1}{2}(p+q)$ are positive:

$$\cot p\cot q \ge \cot^2\tfrac{1}{2}(p+q)$$ **M1 A1 A.G.**

Since α and β satisfy the conditions for p and q:

$$\cot^2\varphi = \cot\alpha\cot\beta \ge \cot^2\tfrac{1}{2}(\alpha+\beta)$$ **B1**

Since all are positive:

$$\cot\varphi \ge \cot\tfrac{1}{2}(\alpha+\beta)$$ **A1**

Since all angles are acute and *cot* is a decreasing function in this range:

$$\varphi \le \tfrac{1}{2}(\alpha+\beta)$$ **M1 A1 A.G.**

Question 7

(i)
$$x = px + qx \quad \textbf{M1}$$
$$x \neq 0 \Rightarrow p + q = 1 \quad \textbf{A1}$$
(and clearly x can be any non-zero real number.

(ii)
$$x = py + qx \ so \ x(1-q) = py \quad \textbf{B1}$$
$$y = px + qy \ so \ y(1-q) = px \quad \textbf{B1}$$
Since $xy \neq 0$:
$$(1-q)^2 = p^2 \quad \textbf{M1 A1}$$
Therefore
$$1 - q = \pm p \quad \textbf{M1 A1}$$
$$q \pm p = 1$$
$$1 - q = p \Rightarrow x = y \ unless \ p = 0 \quad \textbf{A1}$$
$$1 - q = -p \Rightarrow x = -y \ unless \ p = 0$$
So $x = y$ or $x = -y$ unless $p = 0$ and $q = 1$

(iii)
$$z = py + qx$$
$$x = pz + qy$$
$$y = px + qz$$
Eliminating *z* in pairs gives:
$$(1 - pq)x = (p^2 + q)y \quad \textbf{B1}$$
$$(1 - pq)y = (p + q^2)x$$
Since $xy \neq 0$:
$$(1 - pq)^2 = (p^2 + q)(p + q^2) \quad \textbf{A1}$$
Therefore: **A1**
$$p^3 + q^3 + 3pq - 1 = 0 \quad \textbf{A.G.}$$
$$p^3 + q^3 + 3pq - 1 \equiv (p + q - 1)(p^2 + q^2 - pq + p + q + 1) \quad \textbf{M1 A1}$$
Either $p + q = 1$ or $(p - q)^2 + (p + 1)^2 + (q + 1)^2 = 0$ **A1**

A.G

If $p + q = 1$:
$$\big(1 - p(1 - p)\big)x = (p^2 + 1 - p)y \quad \textbf{M1 A1}$$
$p^2 - p + 1 \equiv \left(p - \frac{1}{2}\right)^2 + \frac{3}{4} \geq 0$, so $x = y$.
$$z = py + qx = (p + q)x = x \quad \textbf{A1}$$
$(p - q)^2 + (p + 1)^2 + (q + 1)^2 = 0$ can only be true if $p = q = -1$. **B1**
$$z = -x - y \Rightarrow x + y + z = 0$$
Therefore, either $x = y = z$ or $x + y + z = 0$ **A1**

A.G.

Question 8

(i)

$$y = xv \Rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v$$ **M1 A1**

The differential equation becomes

$$x^3 v\frac{dv}{dx} + x^2v^2 + x^2v^2 - 2x^2 = 0$$ **M1 A1**

Since $x \neq 0$:

$$xv\frac{dv}{dx} + 2v^2 - 2 = 0$$ **A1** **A.G.**

Separating the variables:

$$\int \frac{v}{1-v^2}\,dv = \int \frac{2}{x}\,dx$$ **M1 A1**

$$-\tfrac{1}{2}\ln|1-v^2| = 2\ln|x| + k$$ **M1 A1**

$$-2\mathrm{k} = 4\ln|x| + \ln|1-v^2|$$

$$x^4 - x^4v^2 = k'$$ **M1 A1**

$$x^2(y^2 - x^2) = C$$ **A1** **A.G.**

Using the substitution $y = xv$ and simplifying gives:

$$xv\frac{dv}{dx} + v^2 + 5v + 6 = 0$$ **M1 A1**

$$\int \frac{v}{(v+2)(v+3)}\,dv = -\int \frac{1}{x}\,dx$$ **B1**

Using partial fractions:

$$\frac{v}{(v+2)(v+3)} \equiv \frac{-2}{v+2} + \frac{3}{v+3}$$ **M1 A1**

So

$$-2\ln|v+2| + 3\ln|v+3| + \ln|x| = k$$ **M1 A1**

$$\frac{x(v+3)^3}{(v+2)^2} = A$$

Which simplifies to:

$$(y+3x)^3 = A(y+2x)^2$$ **A1**

Question 9

Vertically we have the following (for the complete motion):

$$s = 2.5\ m$$

$$u = -10\sin\theta\ ms^{-1}$$ **B1**

$$a = 10\ ms^{-2}$$

$$t\ is\ required$$

$$2.5 = -10t\sin\theta + 5t^2$$ **M1 A1**

Therefore (as the positive root is required): **A1 (positive root explained)**

$$t = \frac{10\sin\theta + \sqrt{100\sin^2\theta + 50}}{10} = \sin\theta + \sqrt{\sin^2\theta + \frac{1}{2}}$$

$$c = 1 - 2\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{2}(1-c)$$ **B1**

So

$$t = \frac{1}{\sqrt{2}}\left(\sqrt{1-c} + \sqrt{2-c}\right)$$ **M1 A1 A.G.**

The range is $10t\cos\theta$

$$c = 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1}{2}(1+c)$$ **B1**

$$Range = 5\sqrt{2}\sqrt{\frac{1}{2}(1+c)}\left(\sqrt{1-c} + \sqrt{2-c}\right)$$ **M1 A1**

$$R = 5\sqrt{1+c}\left(\sqrt{1-c} + \sqrt{2-c}\right)$$

Differentiate to find maximum range:

$$\frac{dR}{dc} = \frac{1}{2\sqrt{1+c}}\left(\sqrt{1-c} + \sqrt{2-c}\right) + \sqrt{1+c}\left(-\frac{1}{2\sqrt{1-c}} - \frac{1}{2\sqrt{2-c}}\right)$$ **M1 A1 A1**

Setting equal to 0 and simplifying

$$\sqrt{1-c}\sqrt{2-c}\left(\sqrt{1-c} + \sqrt{2-c}\right) = (1+c)\left(\sqrt{1-c} + \sqrt{2-c}\right)$$ **M1 A1**

$$(1+c)^2 = (1-c)(2-c) \Rightarrow c = \frac{1}{5}$$ **M1 A1 A.G.**

When $c = \frac{1}{5}$, range is $5\sqrt{\frac{6}{5}}\left(\sqrt{\frac{4}{5}} + \sqrt{\frac{9}{5}}\right) = 5\sqrt{6}\ m$ **B1**

When $c = 0$ (projected at 45°), range is $5\left(1 + \sqrt{2}\right)\ m$ **B1**

So the extra distance attained is $5\left(\sqrt{6} - \sqrt{2} - 1\right)\ m$ **A1 A.G.**

Question 10

For first stone to be dropped:

Time taken for the stone to hit the water is $\sqrt{\frac{2D}{g}}\ s$ **B1**

The time for the sound to return to me is $\frac{D}{u}$ s **B1**

Total time is therefore

$$\sqrt{\frac{2D}{g}}+\frac{D}{u}$$ **B1**

For the second stone, D is replaced by $D-\delta$, so the time to hear the splash is:

$$\sqrt{\frac{2(D-\delta)}{g}}+\frac{D-\delta}{u}$$ **B1 B1**

Therefore:

$$t=\left(\sqrt{\frac{2D}{g}}+\frac{D}{u}\right)-\left(\sqrt{\frac{2(D-\delta)}{g}}+\frac{D-\delta}{u}\right)=\sqrt{\frac{2D}{g}}-\sqrt{\frac{2(D-\delta)}{g}}+\frac{\delta}{u}$$ **M1 A1 A.G.**

$$\beta=\sqrt{\frac{2D}{g}}-\sqrt{\frac{2(D-\delta)}{g}}$$ **B1**

$$\Rightarrow\frac{1}{\beta}=\frac{g}{2\delta}\left(\sqrt{\frac{2D}{g}}+\sqrt{\frac{2(D-\delta)}{g}}\right)$$ **M1 A1**

So

$$\frac{2\delta}{\beta g}=\sqrt{\frac{2D}{g}}+\sqrt{\frac{2(D-\delta)}{g}}$$ **B1**

$$T=\frac{1}{2}\left(\beta+\frac{2\delta}{\beta g}\right)=\sqrt{\frac{2D}{g}}$$ **M1 A1**

Therefore:

$$D=\tfrac{1}{2}gT^2$$ **A1 A.G.**

If $u=300\ ms^{-1}$, $g=10\ ms^{-2}$, $t=\frac{1}{5}\,s$ and $\delta=10\ m$:

$$\beta=\tfrac{1}{6}$$ **M1 A1**

$$T=\tfrac{1}{12}+6$$ **M1 A1**

$$D=5\left(\tfrac{1}{12}+6\right)^2\approx 185\ m$$ **M1 A1 A.G.**

Question 11

(i) Vertically at P:

$$2T = Mg$$ **B1**

where T is the tension in the string.

Perpendicular to the plane at A:

$$R_A = 5mg\cos\left(\arctan\frac{7}{24}\right) = \frac{24}{5}mg$$ **A1**

where R_A is the normal reaction at A.

Friction will take maximum value since equilibrium is limiting **B1**

Parallel to the plane at A:

$$\frac{1}{2}Mg = 5mg\sin\left(\arctan\frac{7}{24}\right) + \frac{24}{5}\mu mg$$ **M1 A1**

$$\frac{1}{2}Mg = \frac{7}{5}mg + \frac{24}{5}\mu mg$$

Similarly for B:

$$R_B = \frac{9}{5}mg$$ **A1**

$$\frac{1}{2}Mg = \frac{12}{5}mg + \frac{9}{5}\mu mg$$ **M1 A1**

Solving simultaneously:

$$\mu = \frac{1}{3}$$ **M1 A1**

$$M = 6m$$ **A1**

A.G.

(ii) If the tension in the string is T:

For each of A and B, the friction and component of the weight must add to $3mg$ **B1**

A:

$$T - 3mg = 5ma_A$$ **B1**

P:

$$9mg - 2T = 9ma_P$$ **B1**

B:

$$T - 3mg = 3ma_B$$ **B1**

Also:

$$a_A + a_B = 2a_P$$ **B1**

Solving simultaneously:

$$a_A = \frac{3}{22}g$$ **M1 A1 A1**

$$a_B = \frac{5}{22}g$$ **A1**

$$a_P = \frac{2}{11}g$$

Question 12

$$\int \frac{2t}{(1+t^2)^2}\,dt = -\frac{1}{1+t^2} + c$$ **M1 A1**

Therefore:

$$P(T<1) = \frac{1}{2}$$ **B1**

$$P(1<T<2) = \frac{3}{10}$$ **B1**

$$P(2<T<3) = \frac{1}{10}$$ **B1**

So the required probability is: **M1 A1 A1**

$$\frac{1}{2}(1-p)^2p + \frac{3}{10}(1-p)p + \frac{1}{10}p$$

$$= \frac{1}{10}p(5p^2-13p+9)$$ **A1**

A.G.

Let D be the event that the fire extinguisher is destroyed after 3 years.

$$P(1<T<1.5) = \frac{5}{26}$$ **M1 A1**

$$P(T<1.5 \cap D) = \frac{1}{2}(1-p)^2p + \frac{5}{26}(1-p)p$$ **M1 A1**

$$P(T<1.5|D) = \frac{P(T<1.5 \cap D)}{P(D)}$$ **B1**

Therefore:

$$P(T<1.5|D) = \frac{\frac{1}{2}(1-p)^2p + \frac{5}{26}(1-p)p}{\frac{1}{10}p(5p^2-13p+9)}$$ **M1 A1 A1**

$$P(T<1.5|D) = \frac{5(1-p)(13(1-p)+5)}{13(5p^2-13p+9)} = \frac{5(1-p)(18-13p)}{13(5p^2-13p+9)}$$ **M1 A1 A1**

Question 13

	There are a total of 5^5 possibilities	**B1**
	Number of possibilities with 5 different digits is $$5! = 120$$	**M1 A1**
	Number of possibilities with 4 different digits is $$10 \times 3! \times 4 \times 5 = 1200$$	***Up to four marks for calculation for 4 digits***
e.g.	*5 choices for the missing digit*	**B1**
	4 choices for the digit that is repeated	**B1**
	10 choices for the positions of the repeated digits	**B1**
	3! choices for the order of the remaining digits	**B1**
	Number of possibilities with 3 different digits is $$10 \times (3 \times 10 \times 2 + 5 \times 3 \times 6) = 1500$$	***Up to six marks for calculation for 3 digits***
e.g.	*10 choices for the 3 digits used.*	
	If one digit is used 3 times, then 10 choices for the positions of repeated digits, 3 choices for the digit repeated and 2 choices for the order of the other two digits.	**B1 B1 B1**
	If two digits are repeated once each, then 5 choices for the position of the digit that is not repeated, 3 choices for which digit this is and then 6 choices for the pair of positions for one of the other digits.	**B1 B1 B1**
	Number of possibilities with 2 different digits is $$10 \times (2^5 - 2) = 300$$	**Up to four marks for calculation for 2 digits**
	10 choices of the two digits	**B1**
e.g.	*Each digit has 2 possibilities, so 2^5 in total.*	**B1**
	Two of these only use 1 digit, so subtract 2.	**B1**
	Number of possibilities with 1 digit is $$5$$	**B1**
	Expected number is $$\frac{5 \times 120 + 4 \times 1200 + 3 \times 1500 + 2 \times 300 + 1 \times 5}{3125}$$	**M1 A1**
	Which is 3.3616	**A1 A.G.**